MMA708

MÄLARDALEN UNIVERSITY

THE APPLICATION OF THE FISHER-WEIL DURATION AND CONVEXITY

Participants
Tom
Sylvester
&Qing Fu

Lecturer
Jan R.M Röman

Department of Mathematics and Physics
Mälardalen University
SE-721 23 Västerås, Sweden

Abstract

The price of a bond is a function of the promised payments and the market required rate of return. Since the promised payments are fixed, bond prices change in response to changes in the market. For investors who hold bonds, the issue of sensitivity of a bond's price to changes in the required rate of return is important. There are some measures of bond price sensitivity that are commonly used. They are Macaulay Duration (effective maturity), Modified Duration and Convexity and Fisher-Weil Duration and Convexity. Each of these provides a more exact description of how a bond price changes relative to changes in the required rate of return. In this paper, we will calculate duration and convexity of bonds using the Fisher-Weil duration and convexity method and compare the results with the Macaulay duration.

Table of Contents

1. INTRODUCTION 4

2. FISHER-WEIL DURATION AND CONVEXITY 5

2.1 Formulas and parameters 5

2.2 The application of the Fisher-Weil duration and convexity in EXCEL 5

3. CONCLUSION 7

4. BIBLIOGRAPHY 8

5. APPENDIX .... FEL! BOKMÄRKET ÄR INTE DEFINIERAT.

1. Introduction

As we know, interest rate management has been one of the most important issues in the financial market today. People have been trying to find a specific model or formula for the interest rate management in their bond investments.

In 1938, F.R. Macaulay defined duration when he studied the relationship between bond and interest rate. So, duration and convexity calculation method are named after Macaulay as Macaulay duration. The Macaulay duration can be interpreted as a weighted average time to maturity, which measures the average term of the bond and the sensitivity of the coupon interest rate.

Macaulay duration has two basic assumptions when it is applied in the interest rate management. The first assumption is that yield curve is flat which means the interest rates are the same for different bonds. The second assumption is that the future cash flow is fixed and does not change as the change of interest rate.
Fisher and Weil defined the term structure considering the different rates for the bonds with different yield to maturity in 1971, which is known as Fisher-Weil duration. Convexity can be expressed as the second derivative of price with respect to yield to maturity.

2. Fisher-Weil duration and convexity

2.1 Formulas and parameters

1) The formula of Fisher-Weil duration:

$$D^{FW} = \frac{1}{P}\sum_{i=1}^{n} t_i c_i e^{-y_i t_i} = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-y_i t_i}}{P}\right]$$

Or

$$D^{FW} = \frac{1}{P}\cdot\left[\frac{N\cdot t_n}{(1+y_n)^n} + \sum_{i=1}^{n}\frac{C\cdot t_i}{(1+y_i)^i}\right]$$

Where:

P = the present value,
y = the bond yield,
C = coupon size = $r \cdot P$
N = the nominal amount (or the principal)
n = number of year to maturity.

2) The formula of Fisher-Weil convexity:

$$Cnvx^{FW} = \frac{1}{2P}\sum_{i=1}^{n} t_i^{\,2} c_i e^{-y_i t_i}$$

Or

$$Cnvx^{FW} = \frac{1}{2P}\cdot\left[\frac{N\cdot t_n^{\,2}}{(1+y_n)^n} + \sum_{i=1}^{n}\frac{C\cdot t_i^{\,2}}{(1+y_i)^i}\right]$$

2.2 The application of the Fisher-Weil duration and convexity in EXCEL

For the calculation used to obtain the targeted duration, please see an external excel file. The characteristics of the bonds are however, summarized in the table below.

Name	PV	FW Dur	FW Conv	PV(YTM)	Duration
01M	99,83511	0,083333	0,003472	99,79363	0,083333
02M	99,67598	0,163889	0,01343	99,6025	0,163889
03M	99,49476	0,247222	0,030559	99,3962	0,247222
06M	98,95751	0,497222	0,123615	98,6363	0,497222
09M	98,39184	0,741667	0,275035	97,85272	0,741667
12M	97,66961	0,986111	0,486208	96,93236	0,986111
1037	105,755	0,927778	0,430386	105,3014	0,927778
1040	108,0515	1,590681	1,293033	107,2348	1,590668
1043	106,9486	2,24272	2,620698	106,493	2,243081
1034	117,4586	2,38383	3,004007	116,7726	2,384607
1048	104,8758	2,99885	4,734166	104,4013	2,999312
1045	109,33	4,051911	8,800584	109,1315	4,055052
1046	114,4672	5,119795	14,75684	114,7724	5,128844
1041	121,1466	6,223625	21,99424	122,3007	6,424305
1049	104,7866	7,533318	31,44772	106,1862	7,561255
1050	93,10718	8,548541	39,9104	94,52543	8,576005
1047	115,8979	10,18738	63,97832	117,9812	10,2642

3. Conclusion

With respect to bonds data used in this paper, the Fisher-Weil duration and convexity has proven to provide a slightly lower duration in years compared to conventional duration method. The durations of the zero coupon bonds using Fisher-Weil method converges with the conventional duration method as anticipated and suggested by asset theory. We also obtained higher present values for the bonds using Fisher-Weil method, which corresponds to the belief that investors will be interested in bonds with shorter duration.

4. Bibliography

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